DO NOW: SOLVE USING SUBSTITUTION OR ELIMINATION

$$\begin{cases} 3x - 6y = 24 \\ 5x + 4y = 12 \end{cases}$$

$x = 2y + 8$

$5(2y + 8) + 4y = 12$

$10y + 40 + 4y = 12$

$14y = -28$

$y = -2$

$x = 2(-2) + 8$

$x = 4$

solution

$(4, -2)$

SECTION 11.3B

CRAMER'S RULE

(2X2 & 3X3 MATRIX)

HOMEWORK

P.744 #15-17, 34-36

Cramer's Rule

$$\begin{cases} ax + by = s \\ cx + dy = t \end{cases} \qquad D = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$D_x = \begin{vmatrix} s & b \\ t & d \end{vmatrix} \qquad D_y = \begin{vmatrix} a & s \\ c & t \end{vmatrix}$$

if $D \neq 0$.

$$x = \frac{D_x}{D}, \qquad y = \frac{D_y}{D}$$

Cramer's Rule for Two Equations Containing Two Variables

The solution to the system of equations

$$\begin{cases} ax + by = s \\ cx + dy = t \end{cases}$$

is given by

$$x = \frac{\begin{vmatrix} s & b \\ t & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}, \qquad y = \frac{\begin{vmatrix} a & s \\ c & t \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

provided that

$$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \neq 0$$

$$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$x = \frac{D_x}{D}, \qquad y = \frac{D_y}{D}$$

2x - 3y = 16
5x + 4y = 11

$$D = \begin{vmatrix} 2 & -3 \\ 5 & 4 \end{vmatrix}$$

$$D_x = \begin{vmatrix} 16 & -3 \\ 11 & 4 \end{vmatrix}$$

$$D_y = \begin{vmatrix} 2 & 16 \\ 5 & 11 \end{vmatrix}$$

7x - 2y = 12
-2x + 9y = 21

$$D = \begin{vmatrix} 7 & -2 \\ -2 & 9 \end{vmatrix}$$

$$D_x = \begin{vmatrix} 12 & -2 \\ 21 & 9 \end{vmatrix}$$

$$D_y = \begin{vmatrix} 7 & 12 \\ -2 & 21 \end{vmatrix}$$

2x - 5y = 54
7x + 3y = 24

$$D = \begin{vmatrix} 2 & -5 \\ 7 & 3 \end{vmatrix}$$

$$D_x = \begin{vmatrix} 54 & -5 \\ 24 & 3 \end{vmatrix}$$

$$D_y = \begin{vmatrix} 2 & 54 \\ 7 & 24 \end{vmatrix}$$

EXAMPLE

Solving a System of Linear Equations Using Determinants

Use Cramer's Rule, if applicable, to solve the system

$$\begin{cases} 3x - 6y = 24 \\ 5x + 4y = 12 \end{cases}$$

$$x = \frac{\begin{vmatrix} 24 & -6 \\ 12 & 4 \end{vmatrix}}{\begin{vmatrix} 3 & -6 \\ 5 & 4 \end{vmatrix}} = \frac{(96 - {}^-72)}{(12 - {}^-30)} = \frac{168}{42} = 4$$

$x = 4$

$$y = \frac{\begin{vmatrix} 3 & 24 \\ 5 & 12 \end{vmatrix}}{\begin{vmatrix} 3 & -6 \\ 5 & 4 \end{vmatrix}} = \frac{(36 - 120)}{(12 - {}^-30)} = \frac{-84}{42} = 2$$

Solution $(4, 2)$

$(4, -2)$

$$x = \frac{D_x}{D}, \qquad y = \frac{D_y}{D}$$

Use Cramer's Rule, if applicable, to solve the system

$$x - 3y = 4$$
$$5x + 7y = 8$$

$$x = \frac{D_x}{D}, \qquad y = \frac{D_y}{D}$$

4 Use Cramer's Rule to Solve a System of Three Equations Containing Three Variables

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = c_1 \\ a_{21}x + a_{22}y + a_{23}z = c_2 \\ a_{31}x + a_{32}y + a_{33}z = c_3 \end{cases} \qquad D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0$$

Cramer's Rule for Three Equations Containing Three Variables

$$x = \frac{D_x}{D} \qquad y = \frac{D_y}{D} \qquad z = \frac{D_z}{D}$$

$$D_x = \begin{vmatrix} c_1 & a_{12} & a_{13} \\ c_2 & a_{22} & a_{23} \\ c_3 & a_{32} & a_{33} \end{vmatrix} \qquad D_y = \begin{vmatrix} a_{11} & c_1 & a_{13} \\ a_{21} & c_2 & a_{23} \\ a_{31} & c_3 & a_{33} \end{vmatrix} \qquad D_z = \begin{vmatrix} a_{11} & a_{12} & c_1 \\ a_{21} & a_{22} & c_2 \\ a_{31} & a_{32} & c_3 \end{vmatrix}$$

EXAMPLE U… Rule

$$x = \frac{D_x}{D} \qquad y = \frac{D_y}{D} \qquad z = \frac{D_z}{D}$$

Use Cramer's Rule, if applicable, to solve the following system:

$$\begin{cases} x + 2y + z = 1 \\ 3x + 5y + z = 3 \\ 2x + 6y + 7z = 1 \end{cases}$$

$$X = \frac{\begin{vmatrix} 1 & 2 & 1 \\ 3 & 5 & 1 \\ 1 & 6 & 7 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 1 \\ 3 & 5 & 1 \\ 2 & 6 & 7 \end{vmatrix}}$$

$$1\begin{vmatrix} 5 & 1 \\ 6 & 7 \end{vmatrix} - 2\begin{vmatrix} 3 & 1 \\ 1 & 7 \end{vmatrix} + 1\begin{vmatrix} 3 & 5 \\ 1 & 6 \end{vmatrix}$$

$$1(35-6) - 2(21-1) + 1(18-5)$$

$$29 - 40 + 13 = 2$$

$$D_x = 2$$

$$1\begin{vmatrix} 5 & 1 \\ 6 & 7 \end{vmatrix} - 2\begin{vmatrix} 3 & 1 \\ 2 & 7 \end{vmatrix} + 1\begin{vmatrix} 3 & 5 \\ 2 & 6 \end{vmatrix}$$

$$= 1(35-6) - 2(21-2) + 1(18-10)$$

$$29 - 38 + 8 = -1$$

$$D = -1$$

$$X = \frac{2}{-1} = -2$$

EXAMPLE Using Cramer's Rule

Use Cramer's Rule, if applicable, to solve the following system:

$$\begin{cases} x+2y+z=1 \\ 3x+5y+z=3 \\ 2x+6y+7z=1 \end{cases}$$

Solution: (-2,2,5)

$D = -1$

$y = \dfrac{\begin{vmatrix} 1 & 1 & 1 \\ 3 & 3 & 1 \\ 2 & 1 & 7 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 1 \\ 3 & 5 & 1 \\ 2 & 6 & 7 \end{vmatrix}}$

$1\begin{vmatrix} 3 & 1 \\ 1 & 7 \end{vmatrix} - 1\begin{vmatrix} 3 & 1 \\ 2 & 7 \end{vmatrix} + 1\begin{vmatrix} 3 & 3 \\ 2 & 1 \end{vmatrix}$

$1(21-1) - 1(21-2) + 1(3-6)$

$20 - 19 - 3 \Rightarrow D_y = -2$

$y = \dfrac{2}{1}$

$z = \dfrac{\begin{vmatrix} 1 & 2 & 1 \\ 3 & 5 & 3 \\ 2 & 6 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 1 \\ 3 & 5 & 1 \\ 2 & 6 & 7 \end{vmatrix}}$

$1\begin{vmatrix} 5 & 3 \\ 6 & 1 \end{vmatrix} - 2\begin{vmatrix} 3 & 3 \\ 2 & 1 \end{vmatrix} + 1\begin{vmatrix} 3 & 5 \\ 2 & 6 \end{vmatrix}$

$1(5-18) - 2(3-6) + 1(18-10)$

$-13 + 6 + 2 \Rightarrow D_z = -5$

$z = \dfrac{5}{1}$

Use Cramer's Rule,

$$3x + y + z = 3$$
$$2x + 2y + 5z = -1$$
$$x - 3y - 4z = 2$$

$$x = \frac{D_x}{D} \qquad y = \frac{D_y}{D} \qquad z = \frac{D_z}{D}$$

Cramer's Rule with Inconsistent or Dependent Systems

- If $D = 0$ and at least one of the determinants D_x, D_y, or D_z is different from 0, then the system is inconsistent and the solution set is $\varnothing$ or $\{ \ \}$.
- If $D = 0$ and all the determinants D_x, D_y, and D_z equal 0, then the system is consistent and dependent so that there are infinitely many solutions. The system must be solved using row reduction techniques.