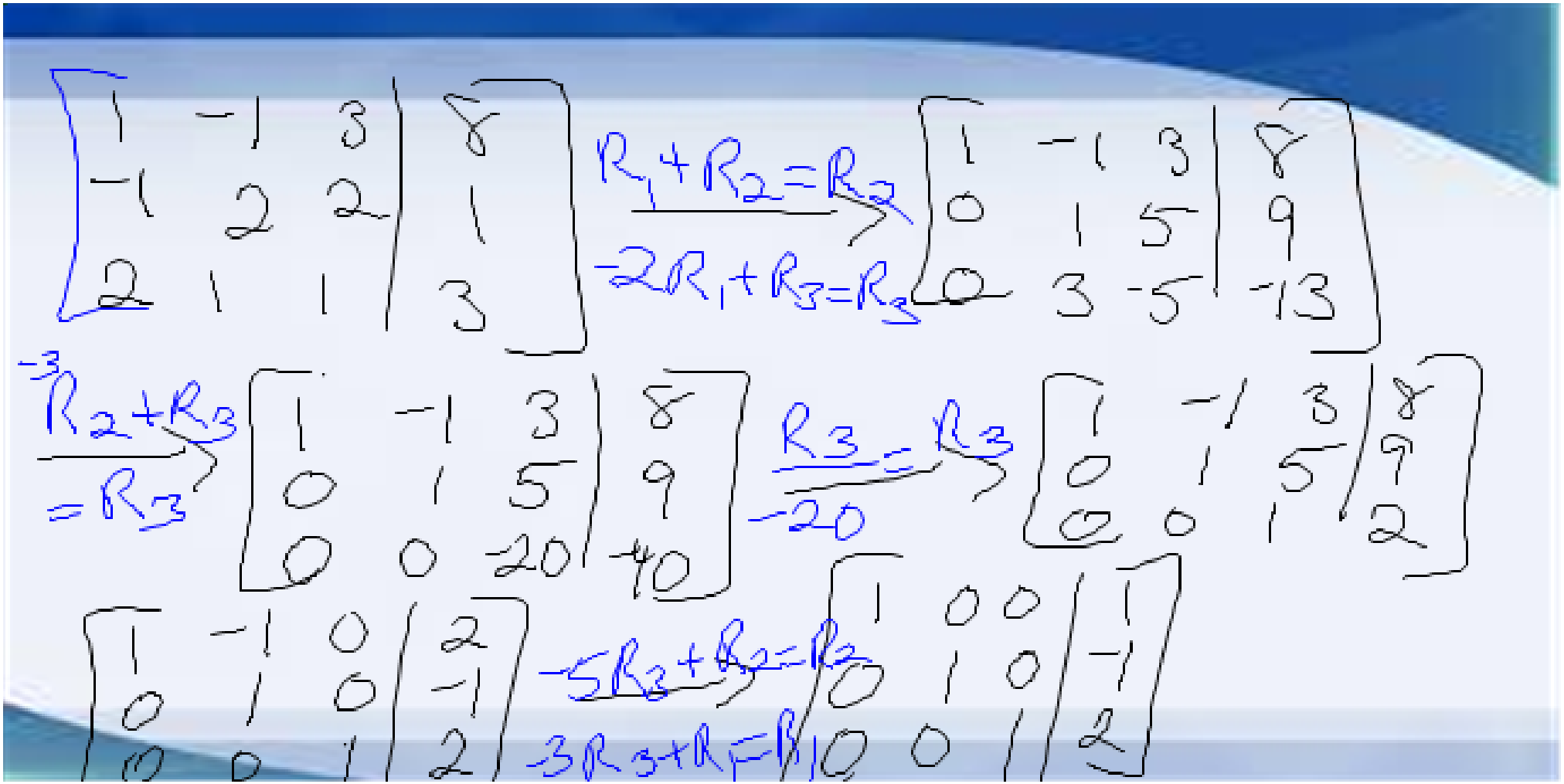
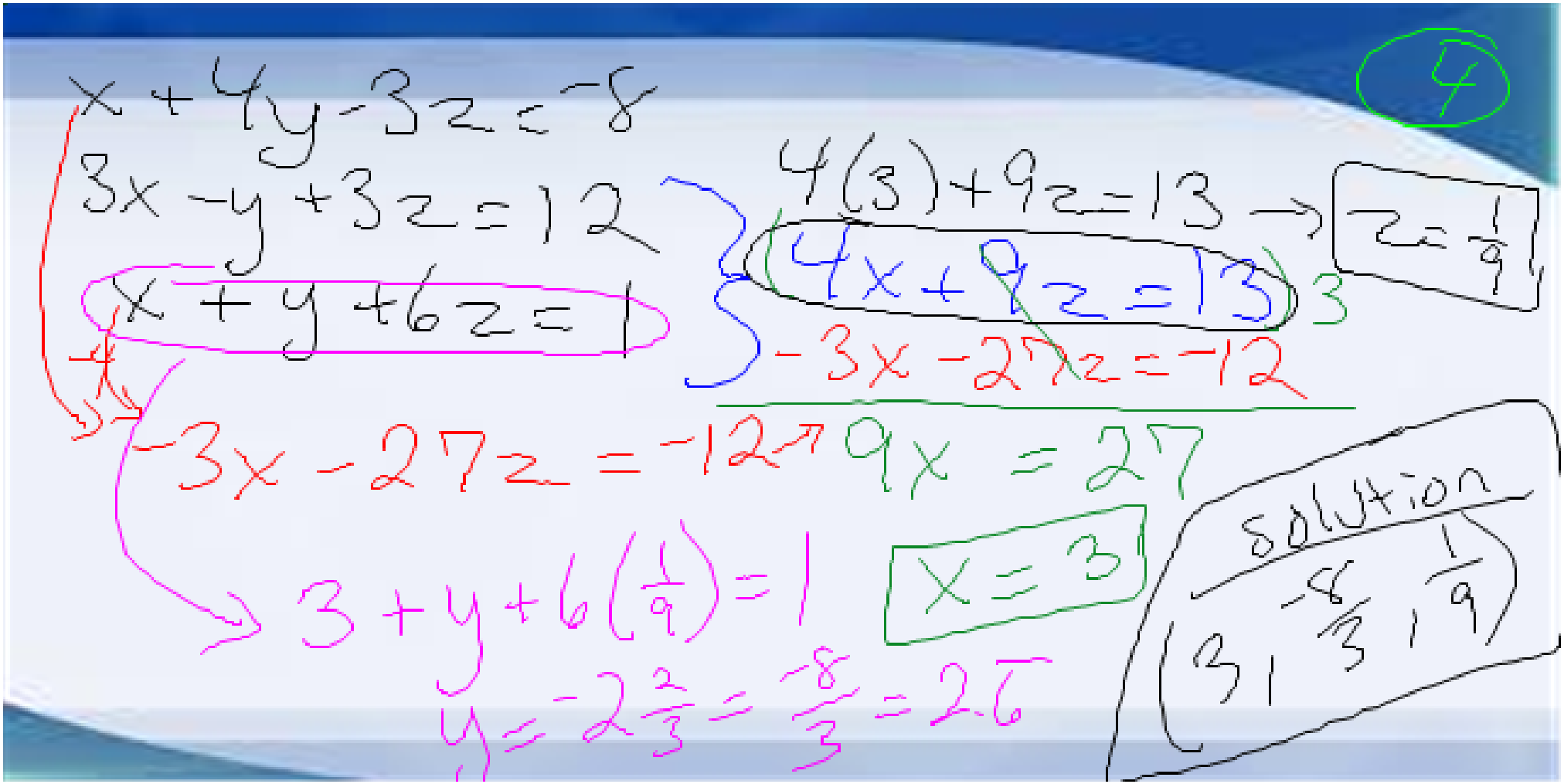
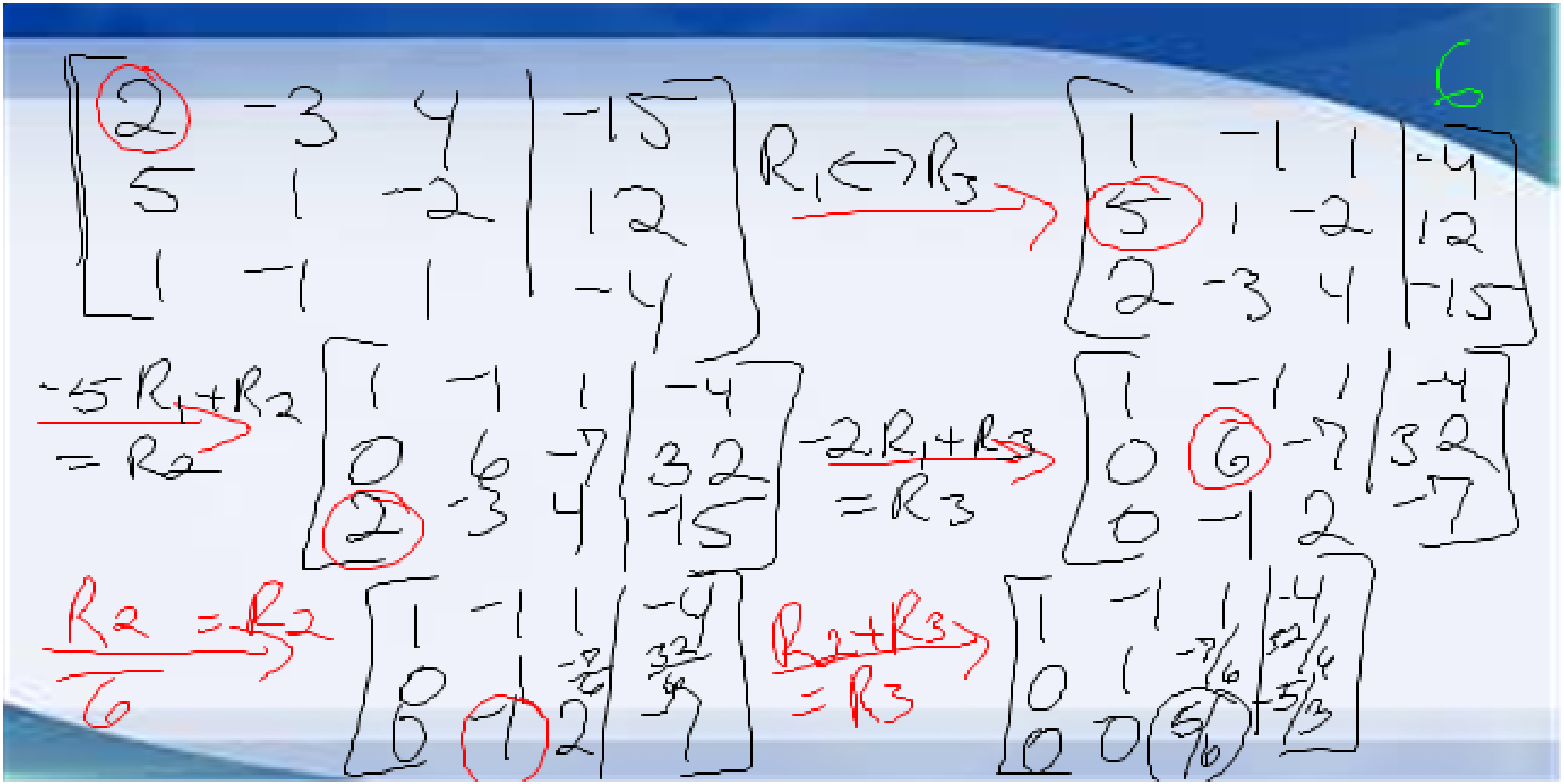
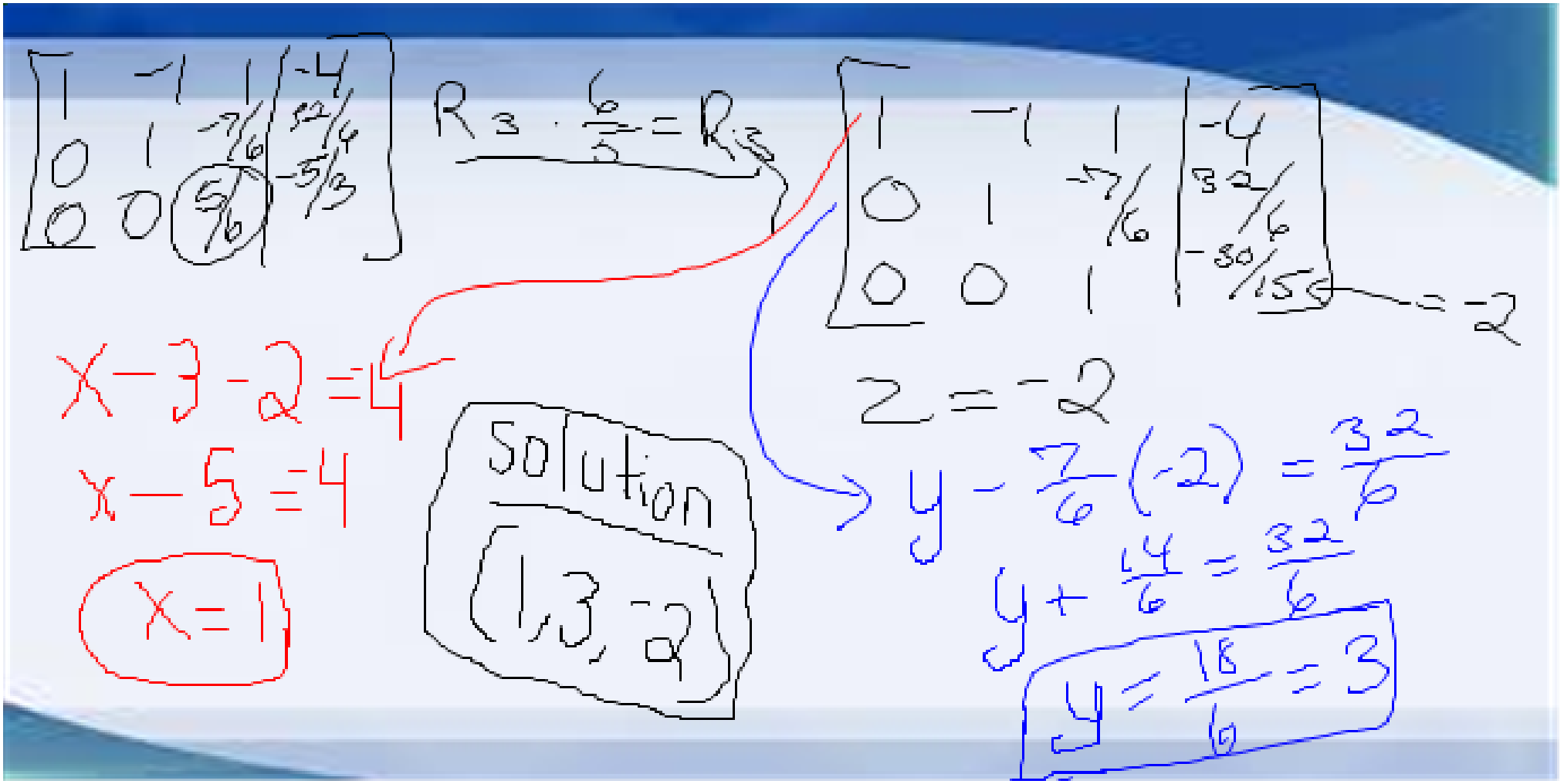

$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 8 \\ -1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 3 \end{array}\right]$
$R_1 + R_2 = R_2$
$-2R_1 + R_3 = R_3$
$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 8 \\ 0 & 1 & 5 & 9 \\ 0 & 3 & -5 & -13 \end{array}\right]$
$-3R_2 + R_3 = R_3$
$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 8 \\ 0 & 1 & 5 & 9 \\ 0 & 0 & -20 & -40 \end{array}\right]$
$\frac{R_3}{-20} = R_3$
$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 8 \\ 0 & 1 & 5 & 9 \\ 0 & 0 & 1 & 2 \end{array}\right]$
$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right]$
$-5R_3 + R_2 = R_2$
$3R_3 + R_1 = R_1$
$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right]$


4
x + 4y - 3z = -8
3x - y + 3z = 12
x + y + 6z = 1
4(3) + 9z = 13 → z = 1/9
4x + 9z = 13
3
-3x - 27z = -12
-3x - 27z = -12 → 9x = 27
x = 3
3 + y + 6(1/9) = 1
y = -2 2/3 = -8/3 = 2.6
solution
(3, -8/3, 1/9)


$R_1 \leftrightarrow R_3$
$-5R_1 + R_2 = R_2$
$-2R_1 + R_3 = R_3$
$\frac{R_2}{6} = R_2$
$R_2 + R_3 = R_3$
6


$\begin{bmatrix} 1 & -1 & 1 & -4 \\ 0 & 1 & -7/6 & 32/6 \\ 0 & 0 & 5/6 & -5/3 \end{bmatrix}$
$R_3 \cdot \frac{6}{5} = R_3$
$\begin{bmatrix} 1 & -1 & 1 & -4 \\ 0 & 1 & -7/6 & 32/6 \\ 0 & 0 & 1 & -30/15 \end{bmatrix} = -2$
$z = -2$
$y - \frac{7}{6}(-2) = \frac{32}{6}$
$y + \frac{14}{6} = \frac{32}{6}$
$y = \frac{18}{6} = 3$
$x - 3 - 2 = -4$
$x - 5 = -4$
$x = 1$
Solution
(1, 3, -2)

SECTION 11.3A

DETERMINANTS

(2X2 & 3X3 MATRIX)

HOMEWORK

P.744 #5-14

DEFINITION

If a, b, c, and d are four real numbers, the symbol

$$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

is called a **2 by 2 determinant**. Its value is the number $ad - bc$; that is,

$$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

EXAMPLE

Evaluating a 2 × 2 Determinant

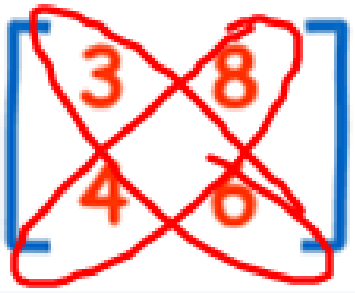
Evaluate: $\begin{vmatrix} -2 & 3 \\ 4 & -1 \end{vmatrix}$ → $(-2)(-1) - (3 \cdot 4)$

$2 - 12$

-10

EXAMPLE

Evaluating a 2 × 2 Determinant

Evaluate: 

$D = -14$

$3 \cdot 6 - 4 \cdot 8$

$18 - 32 = \boxed{-14}$

A **3 by 3 determinant** is symbolized by

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

in which $a_{11}, a_{12}, \ldots,$ are real numbers.

Minus

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

2 by 2 determinant left after removing row and column containing a_{11}

2 by 2 determinant left after removing row and column containing a_{12}

2 by 2 determinant left after removing row and column containing a_{13}

EXAMPLE

Evaluating a 3 × 3 Determinant

Find the value of the 3 by 3 determinant:

$$\begin{vmatrix} 1 & 2 & 1 \\ 3 & 5 & 1 \\ 2 & 6 & 7 \end{vmatrix}$$

+ − +

$= 1\begin{vmatrix} 5 & 1 \\ 6 & 7 \end{vmatrix} - 2\begin{vmatrix} 3 & 1 \\ 2 & 7 \end{vmatrix} + 1\begin{vmatrix} 3 & 5 \\ 2 & 6 \end{vmatrix}$

$1(5 \cdot 7 - 6 \cdot 1) - 2(3 \cdot 7 - 2 \cdot 1) + 1(3 \cdot 6 - 2 \cdot 5)$

$1(35 - 6) - 2(19) + 1(8)$

$29 - 38 + 8 = -1$

Find the value of the 3 by 3 determinant:

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 1 \\ 0 & 3 & -1 \end{bmatrix}$$

$$1\begin{vmatrix} -4 & 1 \\ 3 & -1 \end{vmatrix} - 2\begin{vmatrix} 0 & 1 \\ 0 & -1 \end{vmatrix} + 3\begin{vmatrix} 0 & -4 \\ 0 & 3 \end{vmatrix}$$

$$1(-4(-1) - 1(3)) - 2(0 - 0) + 3(0 - 0)$$

$$1 - 0 + 0 = 1$$

$$\boxed{D = 1}$$

Find the value of the 3 by 3 determinant:

$$\begin{bmatrix} 5 & -2 & 1 \\ 0 & 3 & -1 \\ 2 & 0 & 7 \end{bmatrix}$$