

DO NOW....

Solve this matrix through Reduced Row Echelon Form...

$$\left[ \begin{array}{cc|c} 4 & 8 & 20 \\ -4 & 2 & -30 \end{array} \right] \xrightarrow[\substack{R_1 = R_1 \\ 4}}{\substack{R_1 = R_1 \\ 4}} \left[ \begin{array}{cc|c} 1 & 2 & 5 \\ -4 & 2 & -30 \end{array} \right] \xrightarrow[\substack{R_1 + R_2 \\ = R_2}]{\substack{R_1 + R_2 \\ = R_2}} \left[ \begin{array}{cc|c} 1 & 2 & 5 \\ 0 & 10 & -10 \end{array} \right]$$

$$\xrightarrow[\substack{R_2 = R_2 \\ 10}]{\substack{R_2 = R_2 \\ 10}} \left[ \begin{array}{cc|c} 1 & 2 & 5 \\ 0 & 1 & -1 \end{array} \right] \xrightarrow[\substack{R_2 + R_1 \\ = R_1}]{\substack{R_2 + R_1 \\ = R_1}} \left[ \begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & -1 \end{array} \right]$$

solution  
 $\begin{pmatrix} 7 \\ -1 \end{pmatrix}$

## Solve Through Substitution:

$$\begin{cases} x + y = 6 \\ -2x - 2y = -12 \end{cases} \rightarrow y = -x + 6$$

$$\rightarrow -2x - 2(-x + 6) = -12$$

$$-2x + 2x - 12 = -12$$

$$-12 = -12 \checkmark$$

0  $\neq$  -5  
no solution

Infinite  
solution



# Solving an Inconsistent System of Linear Equations Using Matrices

$$\begin{cases} x + y + z = 6 \\ 2x - y - z = 3 \\ x + 2y + 2z = 1 \end{cases}$$

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & -1 & -1 & 3 \\ 1 & 2 & 2 & 1 \end{array} \right]$$

$$\xrightarrow{-2R_1 + R_2 = R_2}$$

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & -3 & -9 \\ 0 & 2 & 2 & -5 \end{array} \right]$$

$$\xrightarrow{-1R_1 + R_3 = R_3}$$

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & -3 & -9 \\ 0 & 1 & 1 & 5 \end{array} \right]$$

$$\xrightarrow{R_2 = R_2 \cdot (-1/3)}$$

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & 3 \\ 0 & 1 & 1 & 5 \end{array} \right]$$

$$\xrightarrow{-1R_2 + R_3 = R_3}$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & -2 \end{array} \right]$$

$$\xrightarrow{R_2 + R_3 = R_3}$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & -1 \end{array} \right]$$



$$\begin{cases} x = 3 \\ y + z = 3 \\ 0 \neq -1 \end{cases}$$

no solution

Row echelon

$$\left[ \begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & -1 & 3 & 5 \\ 0 & 0 & -1 & 6 \end{array} \right]$$

$3 \times 3$

$$\rightarrow \begin{array}{l} x - y + 2z = 4 \\ -y + 3z = 5 \\ z = 6 \end{array}$$

