

Do Now:

Find the inverse of this matrix...

$$x - y = 3$$

$$\left. \begin{array}{l} x - y = 3 \\ -x + 3y + 4z = -3 \\ 4y + 3z = 2 \end{array} \right\}$$

$$4y + 3z = 2$$

$$A = \begin{bmatrix} 4 & 6 \\ 2 & 4 \end{bmatrix}$$

$$\det(A) = 16 - 12 = 4$$

$$A^{-1} = \frac{1}{\det(A)} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \frac{1}{4} \cdot \begin{bmatrix} 4 & -6 \\ -2 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -\frac{3}{2} \\ -\frac{1}{2} & 1 \end{bmatrix}$$

OBJECTIVE 5

- 5 Solve a System of Linear Equations Using an Inverse Matrix.

EXAMPLE**Using the Inverse Matrix to Solve a System of Linear Equations**

$$A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 3 & 4 \\ 0 & 4 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 \\ -3 \\ 2 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$AX = B$$

$$A^{-1}(AX) = A^{-1}B$$

$$(A^{-1}A)X = A^{-1}B$$

$$I_3X = A^{-1}B$$

$$X = A^{-1}B$$

EXAMPLE

Using the Inverse Matrix to Solve a System of Linear Equations

$$\begin{cases} x + y = 3 \\ -x + 3y + 4z = -3 \\ 4y + 3z = 2 \end{cases} \quad A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 3 & 4 \\ 0 & 4 & 3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -\frac{1}{4} & \frac{1}{4} & -\frac{1}{3} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{3} \\ 0 & -\frac{1}{4} & \frac{1}{3} \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \begin{bmatrix} \frac{1}{4} & \frac{3}{4} & -1 \\ -\frac{1}{4} & \frac{3}{4} & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -3 \\ 2 \end{bmatrix}$$

$$\begin{aligned} &-\frac{1}{4}(3) + \frac{1}{4}(-3) + (-1)(2) \\ &-\frac{1}{4}(3) + \frac{1}{4}(-3) + 2 \end{aligned}$$

$$= \begin{bmatrix} -2 \\ 2 \\ -2 \end{bmatrix}$$
$$3 + -3 + -2$$

EXAMPLE

Using the Inverse Matrix to Solve a System of Linear Equations

Solve the system of equations:
$$\begin{cases} x - y + 2z = 1 \\ -y + 3z = -2 \\ 2x + 2y + z = -1 \end{cases}$$

$$\begin{aligned} x - y &= 3 \\ -x + 3y + 4z &= -3 \\ 4y + 3z &= 2 \end{aligned}$$