

## 5.3

# Use Angle Bisectors of Triangles

**Goal** • Use angle bisectors to find distance relationships.

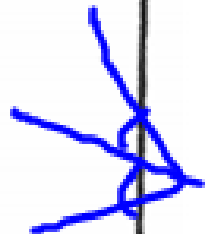


## VOCABULARY

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Incenter

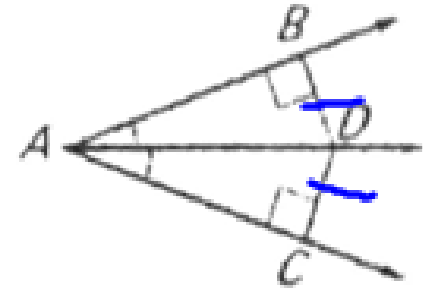
The point of concurrency for Angle bisectors in a triangle



## THEOREM 5.5: ANGLE BISECTOR THEOREM

If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.

If  $\overrightarrow{AD}$  bisects  $\angle BAC$  and  $\overline{DB} \perp \overline{AB}$  and  $\overline{DC} \perp \overline{AC}$ , then  $DB = \underline{CD}$ .

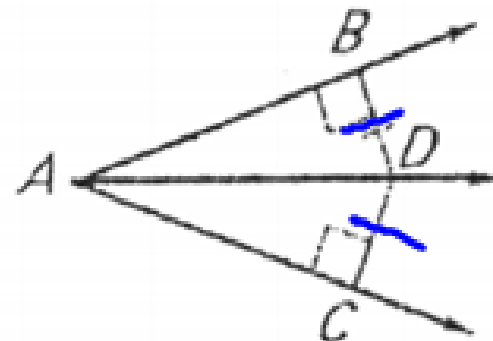


In Geometry,  
*distance* means  
the *shortest* length  
between two  
objects.

## THEOREM 5.6: CONVERSE OF THE ANGLE BISECTOR THEOREM

If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the bisector of the angle.

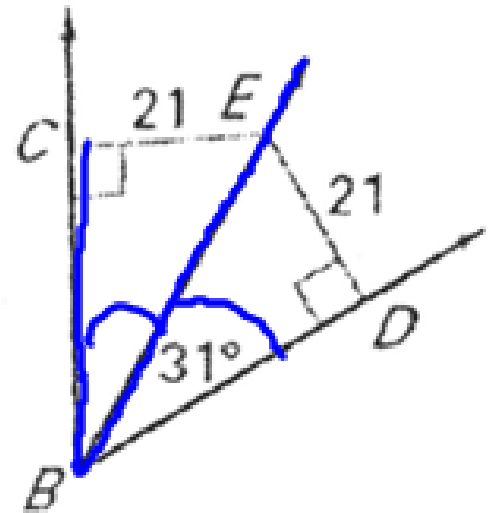
If  $\overline{DB} \perp \overline{AB}$  and  $\overline{DC} \perp \overline{AC}$  and  $DB = DC$ , then  $\overline{AD}$  bisects  $\angle BAC$ .



**Example 1** Use the Angle Bisector Theorems

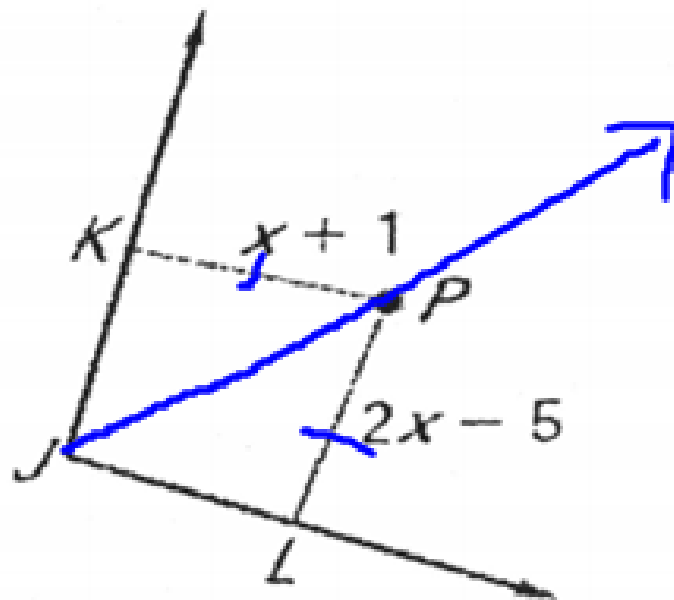
Find the measure of  $\angle CBE$ .

$\angle CBE = 31^\circ$   
b/c Angle bisector  
theorem -



**Example 3***Use algebra to solve a problem***For what value of  $x$  does  $P$  lie on the bisector of  $\angle J$ ?**

$$\begin{array}{r} \cancel{x} + 1 = 2x - 5 \\ -x \qquad -x \\ \hline 1 = x - 5 \\ +5 \qquad +5 \\ \hline \boxed{6 = x} \end{array}$$

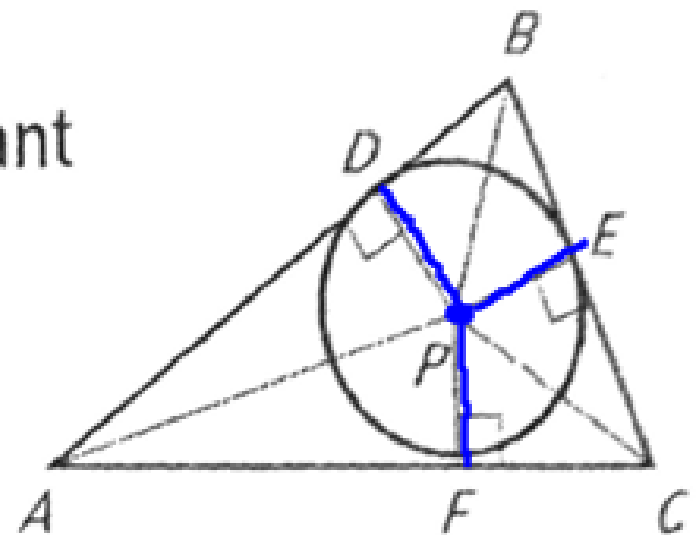


## THEOREM 5.7: CONCURRENCY OF ANGLE BISECTORS OF A TRIANGLE

The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.

If  $\overline{AP}$ ,  $\overline{BP}$ , and  $\overline{CP}$  are angle bisectors of  $\triangle ABC$ , then

$$PD = PE = PF.$$



### Example 4

In the diagram,  $L$  is the Incenter of  $\triangle FHJ$ . Find  $LK$ .

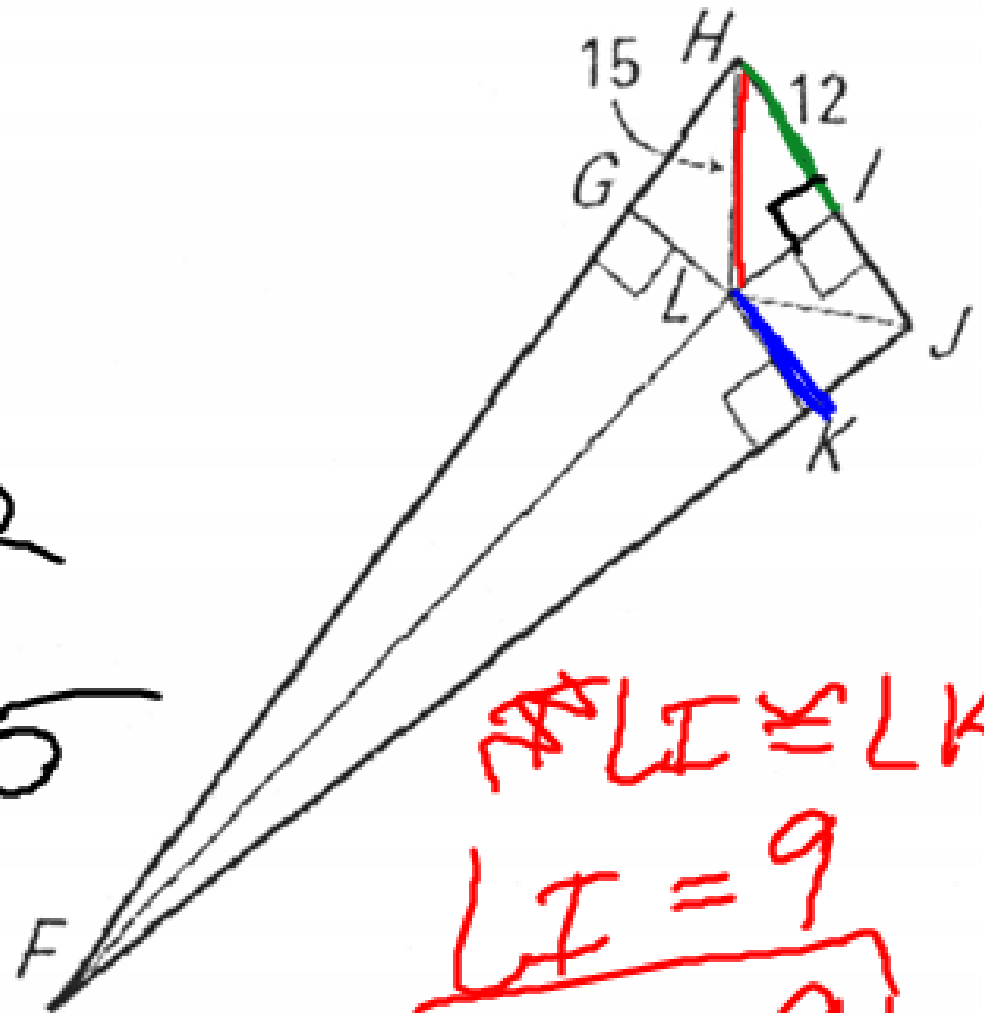
$$a^2 + b^2 = c^2$$

$$12^2 + b^2 = 15^2$$

$$144 + b^2 = 225$$

$$\sqrt{b^2} = \sqrt{81}$$

$$b = 9$$



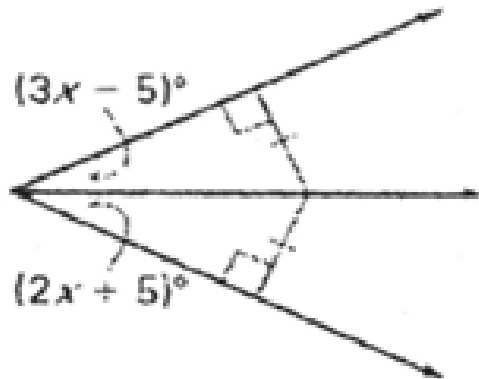
$$\cancel{LI} \neq LK$$

$$LI = 9$$

$$LK = 9$$

✔ **Checkpoint** In Exercises 1 and 2, find the value of  $x$ .

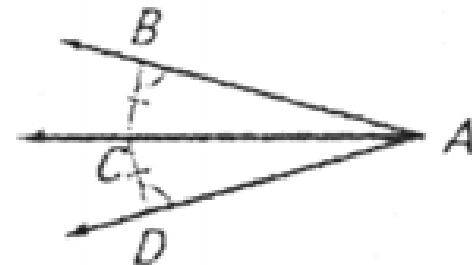
1.



$$3x - 5 = 2x + 5$$
$$\underline{-2x \quad -2x}$$

$$-5 + x = 5$$
$$\underline{+5 \quad +5}$$
$$\boxed{x = 10}$$

3. Do you have enough information to conclude that  $AC$  bisects  $\angle DAB$ ? Explain.



yes b/c angle bisector converse.