Name:

11.1-11.3 Review

$x = \frac{D_x}{D}$ $y = \frac{D_y}{D}$

Solve the system of equations using Cramer's Rule if it is **applicable. If Cramer's Rule is not applicab**le, say so.

$$\begin{cases} 2x + 3y = 11 \\ 3x + 4y = 15 \end{cases}$$

$$x = \frac{\begin{vmatrix} 11 & 3 \\ 15 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix}} = \frac{(44 - 45)}{(8 - 9)} = \frac{-1}{-1} = 1$$

$$y = \frac{\begin{vmatrix} 2 & 11 \\ 3 & 15 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix}} = \frac{(30 - 33)}{(8 - 9)} = \frac{-3}{-1} = 3$$

solution
(1, 3)

Find the value of the determinant.

$$\begin{vmatrix} + & - & + \\ 9 & 0 & 0 \\ 7 & 2 & 4 \\ 7 & 7 & 7 \end{vmatrix}$$

$$= 9\begin{vmatrix} 2 & 4 \\ 7 & 7 \end{vmatrix} - 0\begin{vmatrix} 7 & 4 \\ 7 & 7 \end{vmatrix} + 0\begin{vmatrix} 7 & 2 \\ 7 & 7 \end{vmatrix}$$

$$= 9(14 - 28) - 0 + 0$$

$$D = -126$$

Use Cramer's Rule,

$$x = \frac{D_x}{D} \qquad y = \frac{D_y}{D} \qquad z = \frac{D_z}{D}$$

$3x + y + z = 3$

$2x + 2y + 5z = -1$

$x - 3y - 4z = 2$

$$x = \frac{\begin{vmatrix} 3 & 1 & 1 \\ -1 & 2 & 5 \\ 2 & -3 & -4 \end{vmatrix}}{\begin{vmatrix} 3 & 1 & 1 \\ 2 & 2 & 5 \\ 1 & -3 & -4 \end{vmatrix}}$$

$$\rightarrow 3\begin{vmatrix} 2 & 5 \\ -3 & -4 \end{vmatrix} - 1\begin{vmatrix} -1 & 5 \\ 2 & -4 \end{vmatrix} + 1\begin{vmatrix} -1 & 2 \\ 2 & -3 \end{vmatrix}$$

$3(-8+15) - 1(4-10) + 1(3-4)$

$21 + 6 - 1 = 26$

$$\rightarrow 3\begin{vmatrix} 2 & 5 \\ -3 & -4 \end{vmatrix} - 1\begin{vmatrix} 2 & 5 \\ 1 & -4 \end{vmatrix} + 1\begin{vmatrix} 2 & 2 \\ 1 & -3 \end{vmatrix}$$

$3(-8+15) - 1(-8-5) + 1(-6-2)$

$21 + 13 - 8 = 26$

$x = 1$

Solution

$(1, 1, -1)$

$$Dy = \begin{vmatrix} 3 & 3 & 1 \\ 2 & -1 & 5 \\ 1 & 2 & -4 \end{vmatrix} \rightarrow 3\begin{vmatrix} -1 & 5 \\ 2 & -4 \end{vmatrix} - 3\begin{vmatrix} 2 & 5 \\ 1 & -4 \end{vmatrix} + 1\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix}$$

$3(4-10) - 3(-8-5) + 1(4+1)$

$-18 + 39 + 5 = 26$

$y = 1$

$$Dz = \begin{vmatrix} 3 & 1 & 3 \\ 2 & 2 & -1 \\ 1 & -3 & 2 \end{vmatrix} \rightarrow 3\begin{vmatrix} 2 & -1 \\ -3 & 2 \end{vmatrix} - 1\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} + 3\begin{vmatrix} 2 & 2 \\ 1 & -3 \end{vmatrix}$$

$3(4-3) - 1(4+1) + 3(-6-2)$

$3 - 5 - 24 = -26$

$z = -1$

Solve this matrix using reduced **row echelon form.**

$$\left[\begin{array}{cc|c} 2 & 9 & -7 \\ 1 & 8 & 12 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cc|c} 1 & 8 & 12 \\ 2 & 9 & -7 \end{array}\right] \xrightarrow{-2(R_1)+R_2 = R_2} \left[\begin{array}{cc|c} 1 & 8 & 12 \\ 0 & -7 & -31 \end{array}\right] \xrightarrow{\frac{R_2}{-7} = R_2}$$

$$\left[\begin{array}{cc|c} 1 & 8 & 12 \\ 0 & 1 & \frac{31}{7} \end{array}\right] \xrightarrow{-8(R_2)+R_1 = R_1} \left[\begin{array}{cc|c} 1 & 0 & \frac{22}{7} \\ 0 & 1 & \frac{31}{7} \end{array}\right]$$

Solution

$\left(\frac{22}{7}, \frac{31}{7}\right)$

Determine whether the system corresponding to the **given augmented matrix is consistent or inconsistent.** If it is consistent, give the solution.

$$\left[\begin{array}{ccc|c} 1 & 0 & -7 & 9 \\ 0 & 1 & 8 & -2 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$x - 7z = 9$

$z = 0$

$x - 7(0) = 9$

$x = 9$

$y + 8z = -2$

$y + 8(0) = -2$

$y = -2$

Solution

$(9, -2, 0)$

Solve the system of equations **by substitution.**

$$\begin{cases} x + 7y = -2 \\ 3x + y = 34 \end{cases}$$

$x = -7y - 2$

$3(-7y - 2) + y = 34$

$-21y - 6 + y = 34$

$-20y = 40$

$y = -2$

$x = -7(-2) - 2$

$x = 14 - 2$

$x = 12$

Solution

$(12, -2)$

Solve the system of equations.

$$\begin{cases} x + y + z = 7 \\ x - y + 2z = 7 \\ 5x + y + z = 11 \end{cases}$$

$R_1 + R_2 = (2x + 3z = 14) \cdot -1$

$R_2 + R_3 = +6x + 3z = 18$

$4x = 4$

$x = 1$

$6(1) + 3z = 18$

$6 + 3z = 18$

$3z = 12$

$z = 4$

$x + y + z = 7$

$(1) + y + (4) = 7$

$y + 5 = 7$

$y = 2$

solution

$(1, 2, 4)$

Write the augmented matrix of the given system of equations.

$$\begin{cases} 2x + 9y = 68 \\ -2y = -16 \end{cases}$$

$$\left[\begin{array}{cc|c} 2 & 9 & 68 \\ 0 & -2 & -16 \end{array}\right]$$

Write the system of equations corresponding to the given augmented matrix.

$$\left[\begin{array}{cc|c} 2 & 7 & -10 \\ 2 & 2 & 20 \end{array}\right]$$

$2x + 7y = -10$

$2x + 2y = 20$