

Do Now:

No packet..... Grab a Graphing Calculator

Solve by completing the square

$$x^2 - 4x - 91 = 7$$

$$+91 \quad +91$$

$$x^2 - 4x + 4 = 98 + 4$$

$$x^2 - 4x + 4 = 102$$

$$(x-2)^2 = 102$$

$$x-2 = \pm\sqrt{102}$$

$$+2 \quad +2$$

$$x = 2 \pm \sqrt{102}$$

SECTIONS 3.3B

PROPERTIES OF
QUADRATICS

Quadratic Functions

DEFINITION

A **quadratic function** is a function of the form

$$f(x) = ax^2 + bx + c$$

where a , b , and c are real numbers and $a \neq 0$. The domain of a quadratic function consists of all real numbers.

OBJECTIVE 1

- 1 ✓ Graph a Quadratic Function Using Transformations

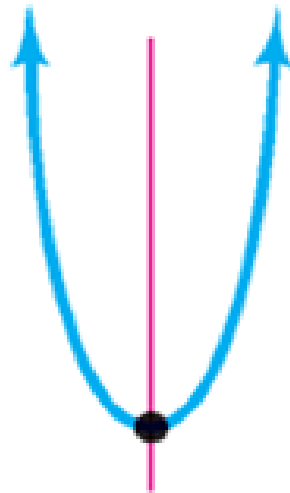
OBJECTIVE 2

- 2 ✓ Identify the Vertex and Axis of Symmetry of a Quadratic Function

On a Calculator...

Graphs of a quadratic function,
 $f(x) = ax^2 + bx + c, a \neq 0$

Axis of
symmetry

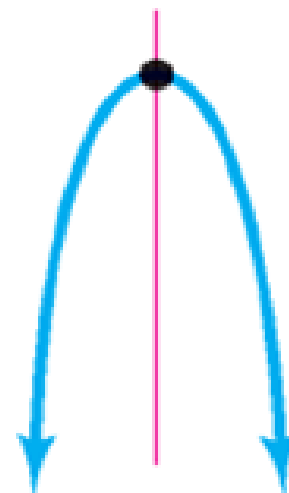


Vertex is
lowest point

(a) Opens up

$$a > 0$$

Vertex is
highest point



Axis of
symmetry

(b) Opens down

$$a < 0$$

EXAMPLE

Graphing a Quadratic Function Using Transformations

Graph the function $f(x) = -2x^2 + 6x + 2$

Find the vertex and axis of symmetry.

$$\text{Vertex: } (1.5, 6.5)$$

$$\text{a.o.s: } x = 1.5$$

Algebraically...

Properties of the Graph of a Quadratic Function

$$f(x) = ax^2 + bx + c \quad a \neq 0$$

$$\text{Vertex} = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right) \right) \quad \text{Axis of symmetry: the line } x = -\frac{b}{2a}$$

Parabola opens up if $a > 0$; the vertex is a minimum point.

Parabola opens down if $a < 0$; the vertex is a maximum point.

EXAMPLE

$$\text{Vertex} = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right) \right)$$

Locating the Vertex without Graphing

Without graphing, locate the vertex and axis of symmetry of the parabola defined by $f(x) = 3x^2 + 12x - 5$. Does it open up or down?

$$a = 3 \quad b = 12 \quad c = -5$$

$$\frac{-b}{2a} = \frac{-12}{2(3)} = \frac{-12}{6} = \boxed{-2 = x}$$

$$V = (-2, 17)$$

$$f(-2) = 3(-2)^2 + 12(-2) - 5$$
$$12 - 24 - 5$$
$$4 = \sqrt{-17}$$

$$\text{a.o.s.} = \boxed{x = -2}$$

EXAMPLE

$$\text{Vertex} = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right) \right)$$

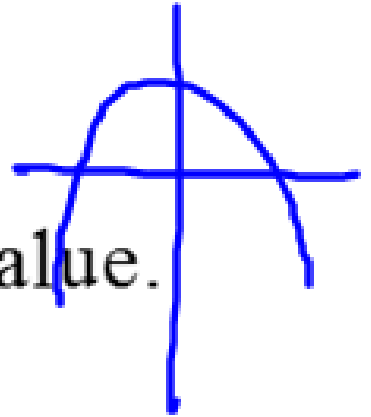
Finding the Maximum or Minimum Value of a Quadratic Function

Determine whether the quadratic function

$$f(x) = -x^2 + 4x + 5$$

has a maximum or minimum value.

Then find the maximum or minimum value.



$$\frac{-b}{2a} = \frac{-4}{2(-1)} = \frac{-4}{-2} = \boxed{2 = x}$$

$$f(2) = -(2)^2 + 4(2) + 5$$

y

$$= -4 + 8 + 5 = \boxed{9}$$

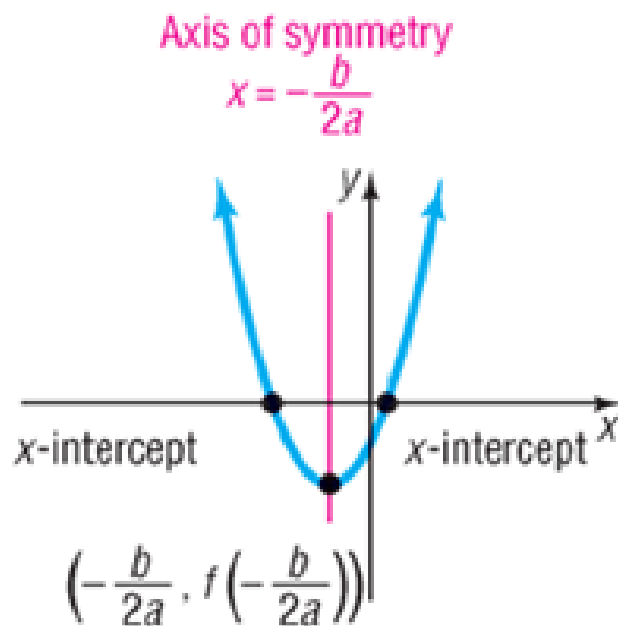
Vertex: $(2, 9)$
a.o.s: $\boxed{x=2}$

OBJECTIVE 3

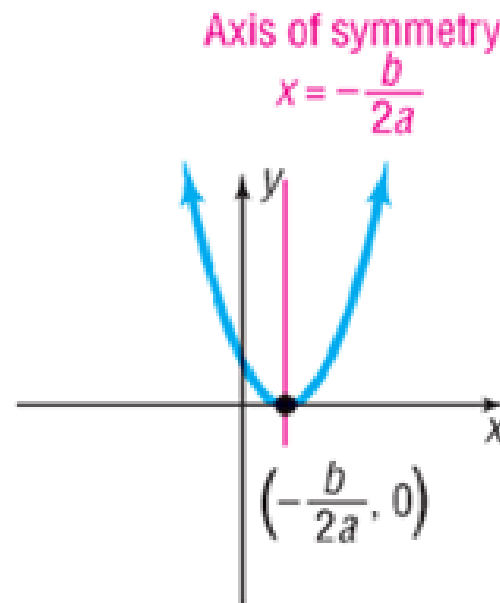
- 3 Graph a Quadratic Function Using Its Vertex, Axis, and Intercepts

The x -Intercepts of a Quadratic Function

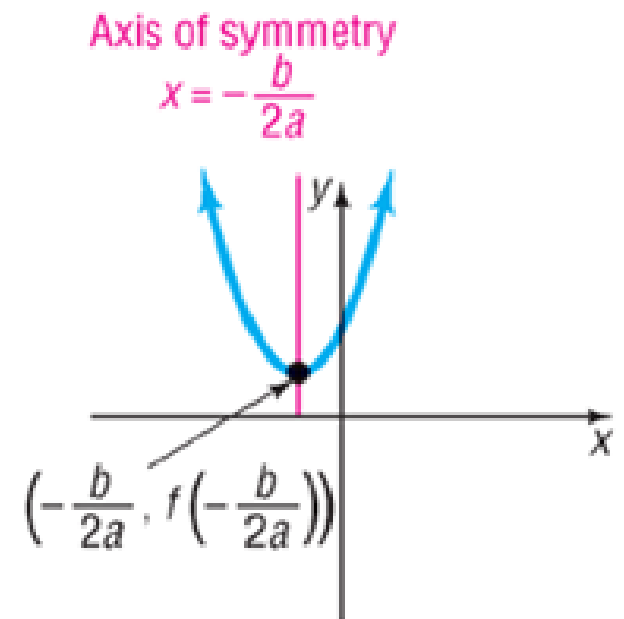
1. If the discriminant $b^2 - 4ac > 0$, the graph of $f(x) = ax^2 + bx + c$ has two distinct x -intercepts so it crosses the x -axis in two places.
2. If the discriminant $b^2 - 4ac = 0$, the graph of $f(x) = ax^2 + bx + c$ has one x -intercept so it touches the x -axis at its vertex.
3. If the discriminant $b^2 - 4ac < 0$, the graph of $f(x) = ax^2 + bx + c$ has no x -intercept so it does not cross or touch the x -axis.



(a) $b^2 - 4ac > 0$
Two x -intercepts



(b) $b^2 - 4ac = 0$
One x -intercept



(c) $b^2 - 4ac < 0$
No x -intercepts

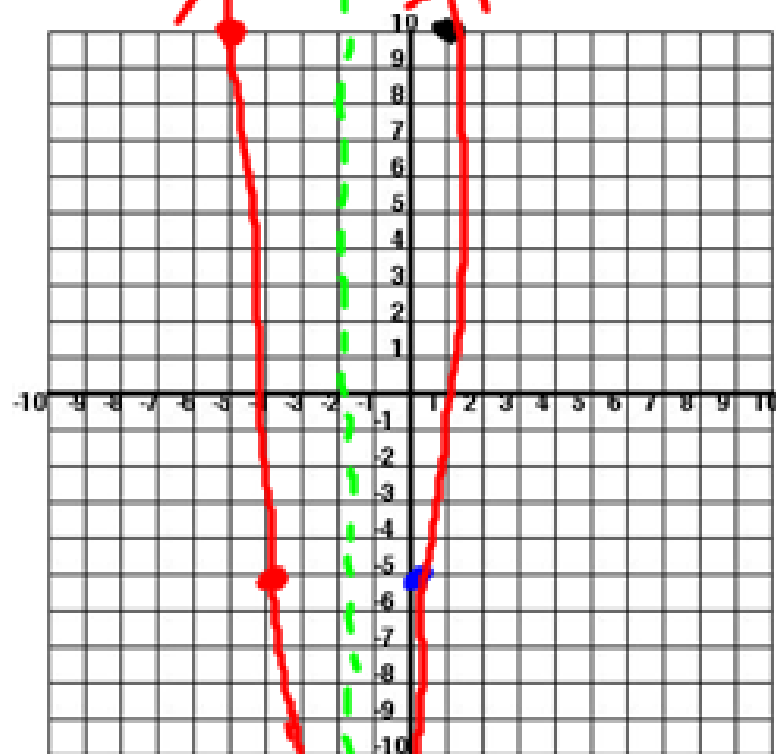
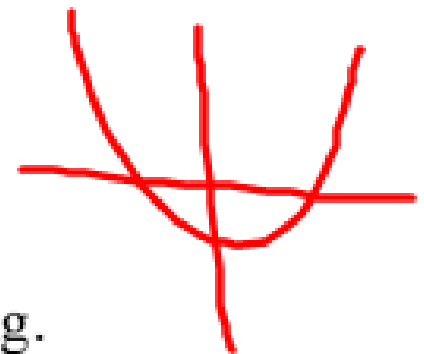
EXAMPLE

How to Graph a Quadratic Function by Hand Using Its Properties

Graph $f(x) = 3x^2 + 12x - 5$ using its properties.

Determine the domain and the range of f .

Determine where f is increasing and where it is decreasing.



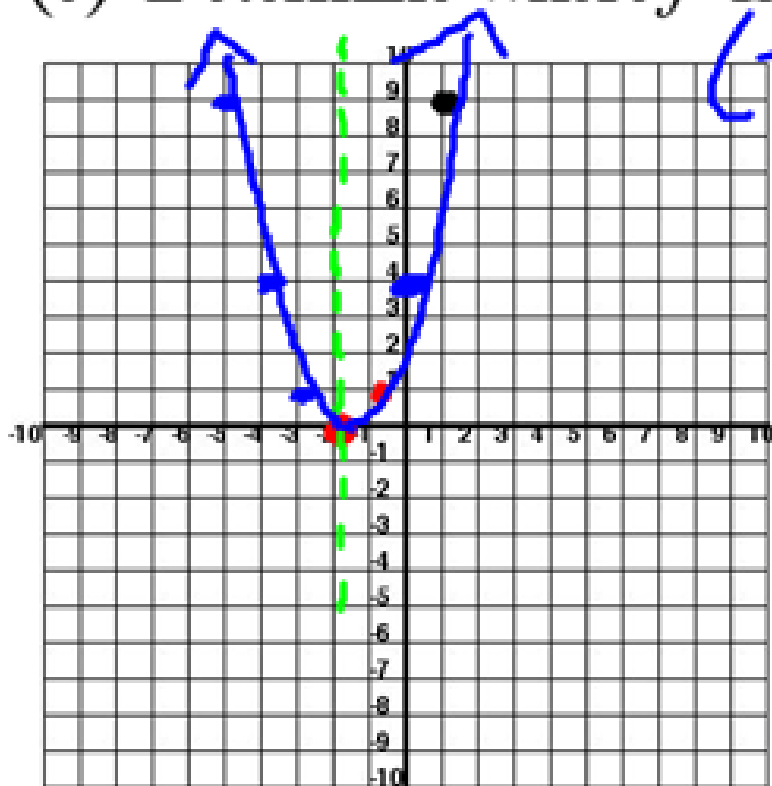
$(-2, -17)$

x	$f(x)$
-1	$3(-1)^2 + 12(-1) - 5$ -14
0	$3(0)^2 + 12(0) - 5$ -5
1	$3(1)^2 + 12(1) - 5$ 10
2	

EXAMPLE

Graphing a Quadratic Function Using Its Vertex, Axis, and Intercepts

- (a) Graph $x^2 + 4x + 4$ by determining whether the graph opens up or down and by finding its vertex, axis of symmetry, y-intercept, and x-intercepts, if any.
- (b) Determine the domain and the range of f .
- (c) Determine where f is increasing and where it is decreasing.



x	$f(x)$
$(-\infty, -2)$	$(-\infty, -2)$
-1	$(-1)^2 + 4(-1) + 4$
0	$0^2 + 4(0) + 4 = 4$
1	$(1)^2 + 4(1) + 4 = 9$

Given a vertex & a pt, find the Quadratic Function...

Given the vertex (h, k) and one additional point on the graph of a quadratic function $f(x) = ax^2 + bx + c$, $a \neq 0$, we can use

$$f(x) = a(x - h)^2 + k \quad (3)$$

to obtain the quadratic function.

EXAMPLE

$$f(x) = a(x - h)^2 + k$$

Finding the Quadratic Function Given Its Vertex and One Other Point

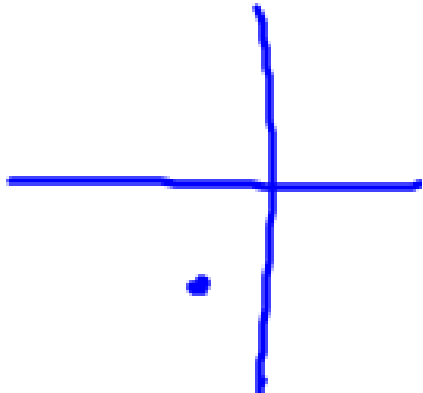
Determine the quadratic function whose vertex is $(-2, -5)$ and whose y intercept is -1 .

$$f(x) = a(x - h)^2 + k$$

$$-1 = a(0 + 2)^2 - 5$$

$$-1 = a(4) + 5$$

$$\begin{array}{r} 4 = a(4) \\ a = 1 \end{array}$$


$$f(x) = 1(x + 2)^2 - 5$$
$$x^2 + 4x + 4 - 5$$
$$x^2 + 4x - 1$$

Describe Transformations from a Quadratic Equation...

$$f(x) = a(x - h)^2 + k$$

$$f(x) = 2x^2 + 8x + 5 = 0$$

~~-5~~ -5

$$\begin{aligned} 2x^2 + 8x &= -5 \\ 2(x^2 + 4x + 4) &= -5 + 8 \\ 2(x + 2)^2 &= 3 \\ \underline{-3} & \quad \underline{-3} \\ 2(x + 2)^2 - 3 &= 0 \end{aligned}$$

SUMMARY Steps for Graphing a Quadratic Function $f(x) = ax^2 + bx + c, a \neq 0$, by Hand

Option 1

STEP 1: Complete the square in x to write the quadratic function in the form $f(x) = a(x - h)^2 + k$.

STEP 2: Graph the function in stages using transformations.

Option 2

STEP 1: Determine whether the graph of f opens up or down.

STEP 2: Determine the vertex $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ and the axis of symmetry, $x = -\frac{b}{2a}$.

STEP 3: Determine the y -intercept, $f(0)$. Determine the x -intercept(s), if any.

- (a) If $b^2 - 4ac > 0$, then the graph of the quadratic function has two x -intercepts, which are found by solving the equation $ax^2 + bx + c = 0$.
- (b) If $b^2 - 4ac = 0$, the vertex is the x -intercept.
- (c) If $b^2 - 4ac < 0$, there are no x -intercepts.

STEP 4: Determine an additional point by using the y -intercept and the axis of symmetry. Plot the points and draw the graph.

EXAMPLE

Graphing a Quadratic Function Using Its Vertex, Axis, and Intercepts

- Graph $-x^2 + 4x + 7$ by determining whether the graph opens up or down and by finding its vertex, axis of symmetry, y -intercept, and x -intercepts, if any.
- Determine the domain and the range of f .
- Determine where f is increasing and where it is decreasing.

