DO NOW ..... Solve:

$$\begin{cases} x + -2y + 3z = 7 \\ 2x + y + z = 4 \\ -3x + 2y - 2z = -10 \end{cases}$$

$R_1 + 2R_2$

$5x + 5z = 15$

$R_1 + R_3$

$-2x + z = -3$

$5x + 5z = 15$

$15x = 30$

$x = 2$

$5(2) + 5z = 15$

$10 + 5z = 15$

$5z = 5$

$z = 1$

$2 + -2y + 3(1) = 7$

$2 - 2y + 3 = 7$

$5 - 2y = 7$

$-2y = 2$

$y = 1$

$x = 2$

$y = -1$

$z = 1$

Worksheet Due Today!

Section 11.2

System Matrices

(Row Echelon Form)

DEFINITION

A matrix is in **row echelon form** when

1. The entry in row 1, column **1** is a 1, and 0's appear below it.
2. The first nonzero entry in each row after the first row is a 1, 0's appear below it, and it appears to the right of the first nonzero entry in any row above.
3. Any rows that contain all 0's to the left of the vertical bar appear at the bottom.

Reduced Row echelon

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$x = 2$
$y = 3$
$z = 4$

$$\left[\begin{array}{ccc|c} 1 & 2 & -4 & 5 \\ 0 & 1 & 6 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

EXAMPLE

How to Solve a System of Linear Equations Using Matrices

$$\begin{cases} 2x + 3y = 1 \\ x - y = 3 \end{cases}$$

$$\left[\begin{array}{cc|c} 2 & 3 & 1 \\ 1 & -1 & 3 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cc|c} 1 & -1 & 3 \\ 2 & 3 & 1 \end{array}\right]$$

$$\xrightarrow{-2R_1 + R_2 = R_2} \left[\begin{array}{cc|c} 1 & -1 & 3 \\ 0 & 5 & -5 \end{array}\right] \xrightarrow{\frac{R_2}{5} = R_2} \left[\begin{array}{cc|c} 1 & -1 & 3 \\ 0 & 1 & -1 \end{array}\right]$$

$$\xrightarrow{R_1 + R_2 = R_1} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \end{array}\right]$$

$$x = 2, \quad y = -1$$

EXAMPLE

How to Solve a System of Linear Equations Using Matrices

$$\begin{cases} x + -2y + 3z = 7 \\ 2x + y + z = 4 \\ -3x + 2y - 2z = -10 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 2 & 1 & 1 & 4 \\ -3 & 2 & -2 & -10 \end{array}\right] \xrightarrow{-2(R_1)+R_2 = R_2}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 0 & 5 & -5 & -10 \\ -3 & 2 & -2 & -10 \end{array}\right] \xrightarrow[= R_3]{3R_1 + R_3}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 0 & 5 & -5 & -10 \\ 0 & -4 & 7 & 11 \end{array}\right]$$

$$\frac{R_2}{5} \Rightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 0 & 1 & -1 & -2 \\ 0 & -4 & 7 & 11 \end{array}\right] \xrightarrow[= R_1]{2R_2 + R_1}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & -1 & -2 \\ 0 & -4 & 7 & 11 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & -1 & -2 \\ 0 & -4 & 7 & 11 \end{array}\right] \xrightarrow{4R_2+R_3 = R_3} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 3 & 3 \end{array}\right]$$

$$\xrightarrow{\frac{R_3}{3} \Rightarrow R_3} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right] \xrightarrow{R_2+R_3 = R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\xrightarrow{-1R_3+R_1 = R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Solving an Inconsistent System of Linear Equations Using Matrices

$$\begin{cases} x + y + z = 6 \\ 2x - y - z = 3 \\ x + 2y + 2z = 1 \end{cases}$$

EXAMPLE

How to Solve a System of Linear Equations Using Matrices

Solve: $\begin{cases} x + y - z = -1 \\ 4x - 3y + 2z = 16 \\ 2x - 2y - 3z = 5 \end{cases}$

Matrix Method for Solving a System of Linear Equations (Row Echelon Form)

Step 1: Write the augmented matrix that represents the system.

Step 2: Perform row operations that place the entry 1 in row 1, column 1.

Step 3: Perform row operations that leave the entry 1 in row 1, column 1 unchanged, while causing 0's to appear below it in column 1.

Step 4: Perform row operations that place the entry 1 in row 2, column 2, but leave the entries in columns to the left unchanged. If it is impossible to place a 1 in row 2, column 2, then proceed to place a 1 in row 2, column 3. Once a 1 is in place, perform row operations to place 0's below it.
[Place any rows that contain only 0's on the left side of the vertical bar, at the bottom of the matrix.]

Step 5: Now repeat Step 4, placing a 1 in the next row, but one column to the right. Continue until the bottom row or the vertical bar is reached.

Step 6: The matrix that results is the row echelon form of the augmented matrix. Analyze the system of equations corresponding to it to solve the original system.

Solving a System of Linear Equations Using Matrices (Row Echelon Form)

Solve: $\begin{cases} x + y + z = 0 \\ -2x + 3y - z = -19 \\ 4x - 3y + 4z = 28 \end{cases}$