Intro to Ch. 11

Solving Systems of Equations

In General, a ***system of equations*** is a collection of two or more equations, each containing 1 or more variables.

EXAMPLE Examples of Systems of Equations

(a) $\begin{cases} 2x + y = 5 & (1) \\ -4x + 6y = -2 & (2) \end{cases}$ Two equations containing two variables, x and y

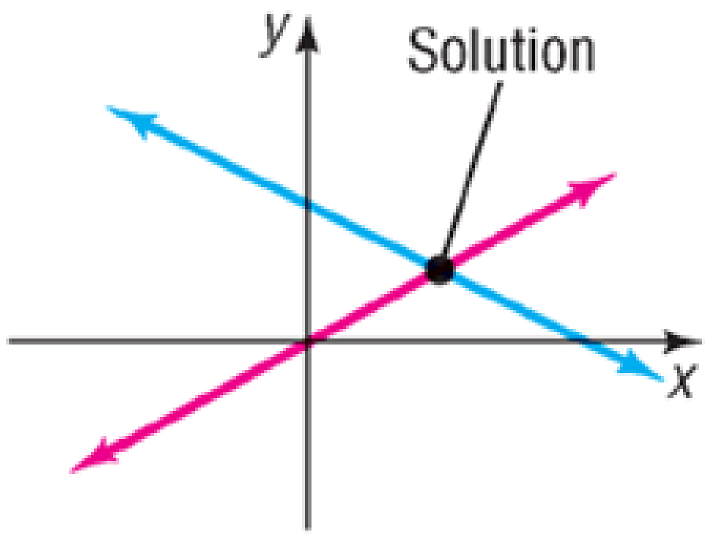
(b) $\begin{cases} x + y^2 = 5 & (1) \\ 2x + y = 4 & (2) \end{cases}$ Two equations containing two variables, x and y

(c) $\begin{cases} x + y + z = 6 & (1) \\ 3x - 2y + 4z = 9 & (2) \\ x - y - z = 0 & (3) \end{cases}$ Three equations containing three variables, x, y, and z

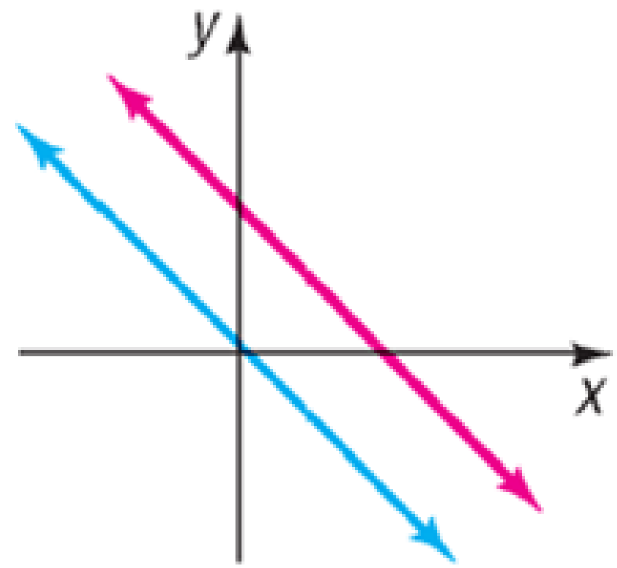
(d) $\begin{cases} x + y + z = 5 & (1) \\ x - y = 2 & (2) \end{cases}$ Two equations containing three variables, x, y, and z

(e) $\begin{cases} x + y + z = 6 & (1) \\ 2x + 2z = 4 & (2) \\ y + z = 2 & (3) \\ x = 4 & (4) \end{cases}$ Four equations containing three variables, x, y, and z

A system of equations is *consistent* is it had at least one solution.



(a) Intersecting lines; system has one solution



(b) Parallel lines; system has no solution

A system is *inconsistent* if it has no solutions.

Two Variables

In a system of two equations with two variables, the equations are *dependent* if one is a multiple of the other.

$x + 2y = 2$

$2x + 4y = 4$

If a system is not dependent then it is independent.

Dependent systems produce the true solution of 0 = 0 & are always consistent

If a system is inconsistent then it produces a false equation of 0 = 5.

Two Variables

Type of System	Example	Nature of Solutions	Graphic
Dependent, Consistent	$x + y = 2$ $\underline{3x + 3y = 6}$ **clue** → $0 = 0$	<u>Infinite</u> number of solutions – they are the same line!	One line "on top of" another 
Independent, Consistent	$x + 2y = 5$ $\underline{-2x + y = 15}$ $x = -5 \quad y = 5$	<u>Unique</u> solution – the lines intersect at one point	Intersection 
Independent, Inconsistent	$2x + 5y = 27$ $\underline{6x + 15y = 39}$ **clue** → $0 = -42$	<u>No</u> solutions – the lines are parallel	

Evaluating a 2-Variable System using the y = mx + b Forms of the Equations

Put the equations into y = mx + b form and examine the slopes and the intercepts:

1. If the **slopes** are the **same** and the **y-intercepts** are the **same**, then the **lines** are the **same** – the system is dependent and consistent.

$$\begin{cases} x + y = 2 & \rightarrow & & & y = -x + 2 \\ 3x + 3y = 6 & \rightarrow & 3y = -3x + 6 & \rightarrow & y = -x + 2 \end{cases}$$

The slope is -1 and the y-intercept is 2 in both equations – **they are the same line** → the system is **dependent and consistent** (infinite number of solutions).

2. If the **slopes** are the **same** but the **y-intercepts** are **different**, then the **lines** are **parallel** (but not the same line) and **will not intersect** – the system is independent and inconsistent.

$$\begin{cases} 2x + 5y = 27 & \rightarrow & y = -\frac{2}{5}x + \frac{27}{5} \\ 6x + 15y = 39 & \rightarrow & y = -\frac{2}{5}x + \frac{39}{15} \end{cases}$$

The slope is $-\frac{2}{5}$ in both equations **but** the y-intercepts are different – these lines are parallel (same slope) but different and will not intersect. The system is **independent and inconsistent** (no solution).

3. If the **slopes** are **different**, it doesn't matter what the y-intercepts are – the lines will always intersect at some point. The system is <u>independent and consistent.</u>

$$\begin{cases} x + 2y = 5 & \rightarrow & y = -\dfrac{2}{5}x + \dfrac{5}{2} \\ -2x + y = 15 & \rightarrow & y = 2x + 15 \end{cases}$$

The slopes are different in the two equations – the lines will intersect. The system is **independent and consistent** (one solution).

Dependent = same line

Independent = different lines

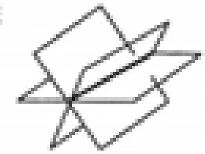
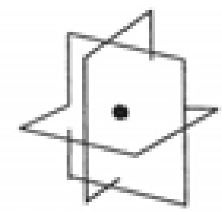
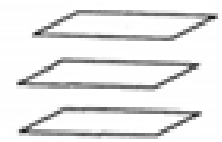
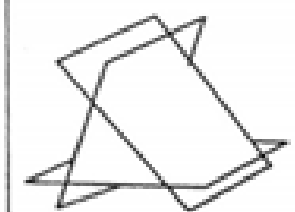
Consistent = solution (different slopes) (same line)

Inconsistent = no solution (same slopes)

Three Variables

In a system of 3 equations with 3 variables, if the system reduces to 2 equations with 2 variables where one is a multiple of another, the system is Dependent.

Three Variables

Type of System	Example	Nature of Solutions	Graphic
Dependent, Consistent	$2x + y + z = 3$ [1] $x - 2y - z = 1$ [2] $3x + 4y + 3z = 5$ [3] [1] + [2] → $3x - y = 4$ [2]*3+[3]→ $\underline{6x - 2y = 8}$ **clue** → $0 = 0$	Infinite number of solutions – the planes intersect along a common line	Intersect along same line 
Independent, Consistent	$4x - 2y - 3z = 5$ $-8x - y + z = -5$ $2x + y + 2z = 5$ $x = 3/2$ $y = -4$ $z = 3$	Unique solution – the planes intersect at one point	Intersect at same point 
Independent, Inconsistent	$y + 3z = 4$ [1] $-x - y + 2z = 0$ [2] $x + 2y + z = 1$ [3] [2] + [3] → $y + 3z = 1$ [4] [1]*(-1)+[4]→ $\underline{-y - 3z = -4}$ **clue** → $0 = -3$	No solution – the planes are either all parallel to each other or they intersect two at a time with no point common to all three	 

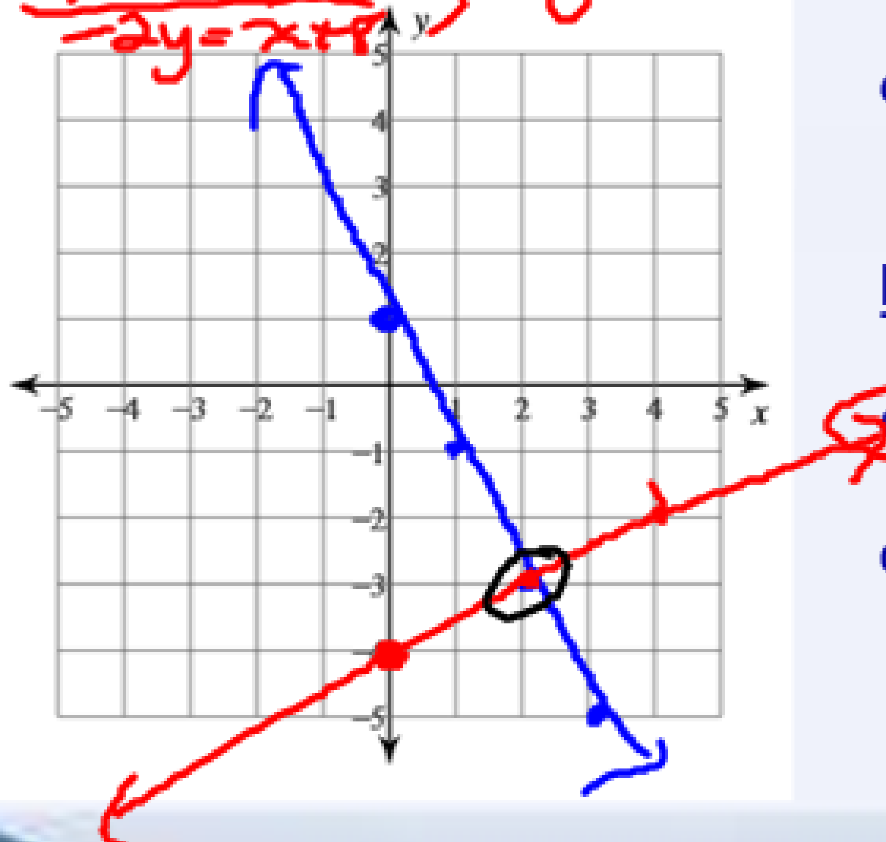
Solve each System Graphically, then answer the Q's below:

1) $2x + y = 1$ → $y = -2x + 1$

$x - 2y = 8$

$-x \qquad -x$

$-2y = -x + 8$ → $y = \frac{1}{2}x - 4$



# of Solutions:

one (2, −3) none infinite

Describe the solutions:

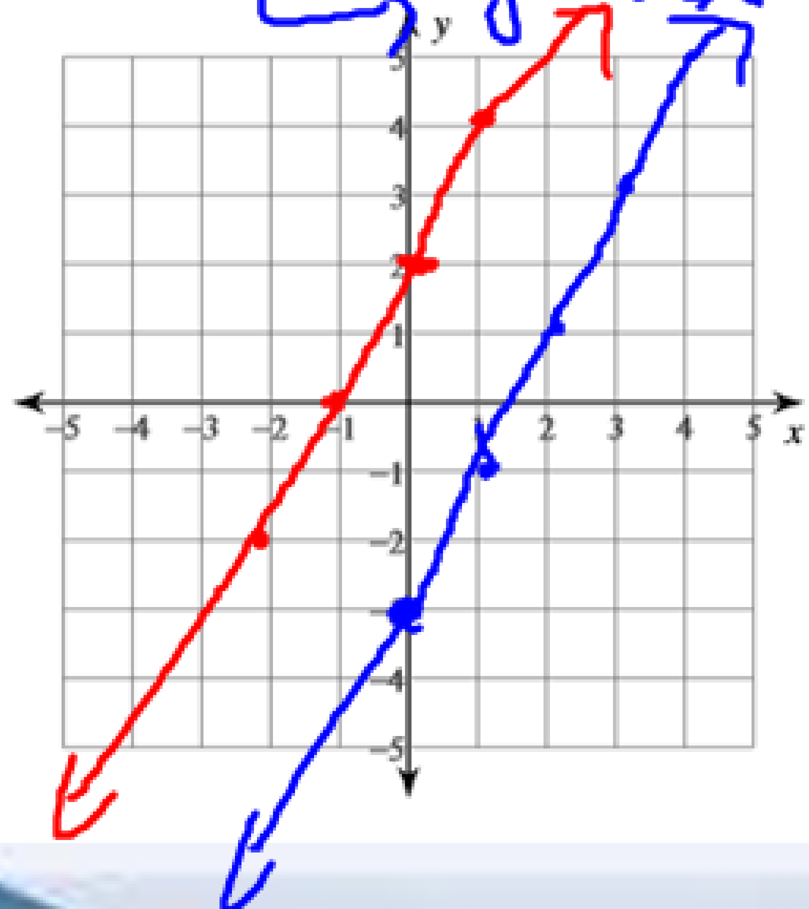
consisent / inconsistent

dependent independent

Solve each System Graphically, then answer the Q's below:

8) $2x - y = -2$ → $y = 2x + 2$

$2x - y = 3$ → $y = 2x - 3$



# of Solutions:

one ________ none ________ infinite

Describe the solutions:

consisent / inconsistent

dependent / independent

Determine if (-1, -2) is a solution of the system below.

12) $x + y = -3$
$x - y = 1$

$x - y = 1$

$(-1) - (-2) = 1$

$-1 + 2 = 1$

$1 = 1$ ✓

(-1, -2) is a solution

Determine if (2, 3) is a solution of the system below.

$x + y = -6$
$4x + y = 3$

$4x + y = 3$

$4(2) + 3 = 3$

$8 + 3 = 3$

$11 \neq 3$ ✗

Not a solution

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